Journal of Statistical Physics, Vol. 42, Nos. 5/6, 1986

The Properties of Connected Graphs and Some Corrections to the List of Uhlenbeck and Ford

J. S. Rowlinson[1]

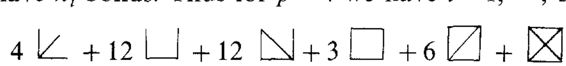
Received August 20, 1985

Some graph-theoretical results for connected cluster integrals are used to correct six entries in the table of Uhlenbeck and Ford.[1]

KEY WORDS: Graph theory; cluster integrals; Uhlenbeck and Ford's list.

Consider a one-dimensional system of hard rods on a line of finite length. The mean number of rods is determined by the chemical potential and can be calculated in two ways. The grand partition function can be expressed as a sum of petit partition functions which stops at the term at which the line is full, or the log grand partition function can be expressed as an infinite series in powers of the activity, the coefficients of which are the connected cluster integrals. A comparison of the two results yields some sum rules for connected graphs which I believe to be new.

Let the distinct graphs with p points be labeled with the index i, and let graph i have n_i bonds. Thus for $p=4$ we have $i=1, —, 38$;



Further, let $n_i^{(2)}$ be the number of unlinked pairs of bonds in graph i, $n_i^{(3)}$, the number of unlinked triplets of bonds, etc. Thus for the six types of graph with $p=4$, $n_i^{(2)}$ takes the successive values of 0, 1, 1, 2, 2, and 3. The graph-theoretical results are

$$\sum_i (-1)^{n_i} = (-1)^{p-1}(p-1)! \qquad (p \geqslant 2)$$

$$\sum_i (-1)^{n_i} n_i = (-1)^{p-1} \tfrac{1}{2} p! \qquad (p \geqslant 2)$$

[1] Physical Chemistry Laboratory, South Parks Road, Oxford, OX1 3QZ, United Kingdom.

0022-4715/86/0300-0931$05.00/0

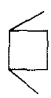
$$\sum_i (-1)^{n_i} n_i^{(2)} = (-1)^{p-1} \tfrac{1}{8}(p-3)\, p! \qquad (p \geqslant 4)$$

$$\sum_i (-1)^{n_i} n_i^{(3)} = (-1)^{p-1} \tfrac{1}{48}(p-4)(p-5)\, p! \qquad (p \geqslant 6)$$

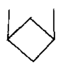
etc.

The standard list of connected graphs with up to six points is that in Appendix 3 of the review of Uhlenbeck and Ford.[1] The results above were used to check this list, and several errors were found. The corrected entries are shown in the table below, in which the first column shows the pair p, n_i, the second is a drawing of the graph, the third is the number of distinct graphs, the fourth is the graph complexity, and the fifth the graph group (these terms are defined by Uhlenbeck and Ford). The entry that has been corrected is underlined.

p. 203, column 2, line 2:

6,6  $\underline{360}$ 4 $S_2(3)$

p. 203, column 2, line 4:

6,6  $\underline{180}$ 4 $S_2(2) \times S_2$

p. 203, column 2, line 8:

6,7  180 8 $S_2 \times S_2 \times E_2$

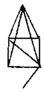
p. 205, column 2, line 8:

6,9  90 64 $\underline{S_2 \times S_2 \times S_2}$

p. 205, column 2, line 13:

6,10  $\underline{120}$ 75 $S_3 \times E_3$

p. 205, column 2, line 14:

6,10  $\underline{180}$ 75 $S_2 \times S_2 \times E_2$

No exhaustive check has been made of the graph complexities or groups, but it is believed that the numbers in column three are now all correct.

REFERENCE

1. G. E. Uhlenbeck and G. W. Ford, in *Studies in Statistical Mechanics*, Vol. 1, Part B. J. de Boer and G. E. Uhlenbeck, eds. (North-Holland, Amsterdam, 1962).

822/42/5-6-14